

# NCERT Solutions Class 8 Maths (Ganita Prakash)

## Chapter 7 Proportional Reasoning 1

### 7.1 Observing Similarity in Change, 7.2 Ratios, 7.3 Ratios in Their Simplest Form, 7.4 Problem Solving with Proportional Reasoning

**Example 1. (i) Are the ratios 3 : 4 and 72 : 96 proportional?**

**(ii) What is the HCF of 72 and 96? (Page 162)**

**Solution:** (i) HCF of 3 and 4 = 1

∴ Ratio 3 : 4 is in its simplest form.

HCF of 72 and 96 = 24 (∵  $72 = 24 \times 3$  and  $96 = 24 \times 4$ )

∴  $72 : 96 = 24 \times 3 : 24 \times 4 = 3 : 4$

∴ Ratio 72 : 96 in its simplest form is 3 : 4.

∴ Ratios 3 : 4 and 72 : 96 are proportional, because both ratios in their simplest form are the same.

(ii) HCF of 72 and 96 is equal to 24.

**Example 2. Kesang wanted to make lemonade for a celebration. She made 6 glasses of lemonade in a vessel and added 10 spoons of sugar to the drink. Her father expected more people to join the celebration. So he asked her to make 18 more glasses of lemonade. To make the lemonade with the same sweetness, how many spoons of sugar should she add? How can we find the factor of change in the ratio? (Page 162)**

**Solution:** Number of glasses of lemonade = 6

Number of spoons of sugar = 10

∴ Ratio of glasses of lemonade to spoon of sugar =  $6 : 10 = 3 : 5$

In the second case, the number of glasses of lemonade = 18

Let the number of spoons of sugar = x

∴ Ratio of glasses of lemonade to spoons of sugar =  $18 : x = 18 : x$

Since sweetness is the same, the ratios 3 : 5 and 18 : x are in proportion.

∴  $35 = 18x$

$\Rightarrow 3x = 5 \times 18 = 90$

$\Rightarrow x = 90 \div 3 = 30$

∴ Number of spoons of sugar = 30.

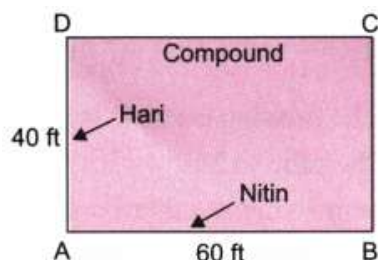
Number of glasses of lemonade in the I case = 6

Number of glasses of lemonade in the II case = 18

∴ Factor of change =  $18 \div 6 = 3$

**Example 3.** Nitin and Hari were constructing a compound wall around their house. Nitin was building the longer side, 60 ft in length, and Hari was building the shorter side, 40 ft in length. Nitin used 3 bags of cement, but Hari used only 2 bags of cement. Nitin was worried that the wall Hari built would not be as strong as the wall he built because he used less cement. Is Nitin correct in his thinking? (Page 163)

**Solution:**



For Nitin, the ratio of the length of the wall to the number of bags of cement =  $60:3=20:1$

For Hari, the ratio of the length of the wall to the number of bags of cement =  $40:2=20:1$

∴ Ratios in simplest form are equal.

∴ Ratios 60 : 3 and 40 : 2 are in proportion.

∴ Walls are equally strong.

∴ Nitin is not correct in his thinking.

Note: Both ratios show that 1 bag of cement is required for a wall of 20 ft.

**Example 4.** In my school, there are 5 teachers and 170 students. The ratio of teachers to students in my school is 5 : 170. Count the number of teachers and students in your school. What is the ratio of teachers to students in your school? Write it below. Is the teacher-to-student ratio in your school proportional to the one in my school? (Page 163)

**Solution:** In the given school, there are 5 teachers and 170 students.

∴ Ratio of teachers to students =  $5:170=1:34$

In my school, there are 6 teachers and 213 students.

∴ Ratio of teachers to students =  $6:213=2:71$

∴ The teacher-to-student ratios 5 : 170 and 6 : 213 are not proportional.

∴ The teacher-to-student ratio in my school is not proportional to that in the given school.

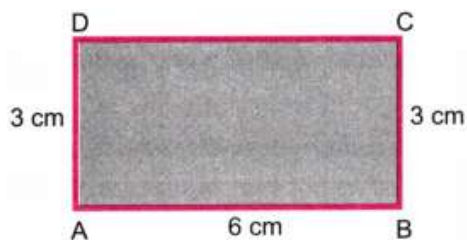
**Example 5.** Measure the width and height (to the nearest cm) of the blackboard in your classroom. What is the ratio of width to height of the blackboard?

Can you draw a rectangle in your notebook whose width and height are proportional to the ratio of the blackboard?

Compare the rectangle you have drawn to those drawn by your classmates. Do they all look the same? (Page 163)

**Solution:** Let the width and height of the blackboard in my classroom be 300 cm and 150 cm, respectively.

∴ Ratio of width to height =  $300\text{ cm}:150\text{ cm}=2:1$



Draw a line AB equal to 6 cm.

Let the height of the rectangle be  $x$  cm, so that the width and height of the rectangle are proportional to the ratio of the blackboard.

$$\therefore 2 : 1 :: 6 : x$$

$$\Rightarrow 21 = 6x$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$\therefore$  The height of the rectangle is 3 cm.

Draw AD and BC perpendicular to AB and of length 3 cm. Join CD.

ABCD is the required rectangle.

Comparing the rectangles drawn by other classmates, I find that the ratios of width and height of all rectangles drawn by other classmates are proportional. Their rectangle are all similar, but they do not look the same.

**Example 6.** When Neelima was 3 years old, her mother was 10 times her age. What is the ratio of Neelima's age to her mother's age? What would be the ratio of their ages when Neelima is 12 years old? Would it remain the same? (Page 164)

**Solution:** Age of Neelima = 3 years

Mother's age is 10 times the age of Neelima.

$$\therefore \text{Age of mother} = 10 \times 3 = 30 \text{ years}$$

$$\therefore \text{Ratio of Neelima's age to her mother's age} = 3 : 30 = 1 : 10$$

Now, Neelima is 12 years old.

Age of Neelima is 12 years, after  $12 - 3 = 9$  years.

After 9 years, age of mother =  $30 + 9 = 39$  years

$$\therefore \text{Ratio of Neelima's age to her mother's age} = 12 : 39 = 4 : 13$$

Remark: It must be noted that ratios  $14 : 13$  are not in proportion.

The rule is that when a fixed number  $k$  is added (or subtracted) from a ratio  $a : b$ , then the new ratio  $a + k : b + k$  may or may not be in proportion with the original ratio  $a : b$ .

In other words,  $a : b :: a + k : b + k$  may not be true.

**Example 7.** Fill in the missing numbers for the following ratios that are proportional to  $14 : 21$ .

(i) \_\_\_\_ : 42

(ii) 6 : \_\_\_\_

(iii) 2 : \_\_\_\_

What factor should we multiply 14 by to get 6? Can it be an integer? Or should it be a fraction? (Page 164)

**Solution:** The given ratio is  $14 : 21$ .

(i) The ratio is \_\_\_\_ : 42.

Let the missing number be x.

$$14 : 21 :: x : 42$$

$$\Rightarrow 14 \times 21 = x \times 42$$

$$\Rightarrow 23 = x \times 42$$

$$\Rightarrow 3x = 2 \times 42 = 84$$

$$\Rightarrow x = 28$$

$\therefore$  The missing number is 28.

(ii) The ratio is 6 : \_\_\_\_

Let the missing number be x.

$$\therefore 14 : 21 :: 6 : x$$

$$\Rightarrow 14 \times 21 = 6x$$

$$\Rightarrow 23 = 6x$$

$$\Rightarrow 2x = 3 \times 6 = 18$$

$$\Rightarrow x = 9$$

$\therefore$  The missing number is 9.

(iii) The ratio is 2 : \_\_\_\_

Let the missing number be x.

$$\therefore 14 : 21 :: 2 : x$$

$$\Rightarrow 14 \times 21 = 2x$$

$$\Rightarrow 23 = 2x$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$\therefore$  The missing number is 3.

Second Part: Ratios under consideration are 14 : 21 and 6 : \_\_\_\_

Now,  $6 \times 14 = 37$  and  $37 \times 14 = 6$

$\therefore$  14 should be multiplied by 37 to get 6.

$\therefore$  Required factor = 37

$\therefore$  Missing term in the ratio 6 : \_\_\_\_ is  $37 \times 21 = 9$ .

$\therefore$  The ratio 6 : \_\_\_\_ is 6 : 9.

The factor here is 37 and this is a fraction.

**Example 8.** Filter coffee is a beverage made by mixing coffee decoction with milk.

Manjunath usually mixes 15 mL of coffee decoction with 35 mL of milk to make one cup of filter coffee in his coffee shop. In this case, we can say that the ratio of coffee decoction to milk is 15 : 35. If customers want 'stronger' filter coffee. Manjunath mixes 20 mL of the decoction with 30 mL of milk. The ratio here is 20 : 30. Why is this coffee stronger?





(i) And when they want 'lighter' filter coffee, he mixes 10 mL of coffee and 40 mL of milk, making the ratio 10 : 40. Why is this coffee lighter?



(ii) The following table shows the different ratios in which Manjunath mixes coffee decoction with milk. Write in the last column if the coffee is stronger or lighter than the regular coffee. (Pages 164-165)

Coffee Decoction (in mL)	Milk (in mL)	Regular/Strong/Light
300	600	
150	500	
200	400	
24	56	
100	300	

**Solution:** In one cup of regular filter coffee: Coffee decoction = 15 mL  
Milk = 35 mL

$\therefore$  Ratio of coffee decoction to milk = 15 : 35 = 3 : 7

Here,  $3 + 7 = 10$

$\therefore$  Coffee decoction in 10 mL filter coffee = 3 mL

$\therefore$  Coffee decoction in 100 mL filter coffee =  $3 \times 10 = 30$  mL

In one cup of stronger filter coffee:

Coffee decoction = 20 mL

Milk = 30 mL

$\therefore$  Ratio of coffee decoction to milk = 20 : 30 = 2 : 3

Here,  $2 + 3 = 5$ .

$\therefore$  Coffee decoction in 5 mL filter coffee = 2 mL

$\therefore$  Coffee decoction in 100 mL filter coffee =  $2 \times 50 = 100$  mL

Since  $100 \text{ mL} > 30 \text{ mL}$ , the latter coffee is stronger.

(i) In a cup of lighter filter coffee:

Coffee decoction = 10 mL

Milk = 40 mL

$\therefore$  Ratio of coffee decoction to milk = 10 : 40 = 1 : 4

Here,  $1 + 4 = 5$ .

$\therefore$  Coffee decoction in 5 mL filter coffee = 1 mL  
 $\therefore$  Coffee decoction in 100 mL filter coffee =  $15 \times 100 = 20$  mL  
 Since  $20 \text{ mL} < 30 \text{ mL}$ , the third type of filter coffee is lighter.  
 (ii)

S. No.	Coffee Decoc-tion (in mL)	Milk (in mL)	Regular/ Stronger/ Lighter
1	300	600	Stronger
2	150	500	Lighter
3	200	400	Stronger
4	24	56	Regular
5	100	300	Lighter

S.No.1. Here  $300 + 600 = 900$   
 $\therefore$  Coffee decoction in 900 mL filter coffee = 300  
 $\therefore$  Coffee decoction in 100 mL filter coffee =  $300900 \times 100$   
 $= 1003$   
 $= 3313 \text{ mL}$   
 Since  $3313 > 30$ , this filter coffee is stronger.  
 S.No.2. Here  $150 + 500 = 650$   
 $\therefore$  Coffee decoction in 650 mL filter coffee = 150  
 $\therefore$  Coffee decoction in 100 mL filter coffee =  $150650 \times 100$   
 $= 30013$   
 $= 23113 \text{ mL}$   
 Since  $23113 < 30$ , this filter coffee is lighter.  
 S.No. 3. Here  $200 + 400 = 600$   
 $\therefore$  Coffee decoction in 600 mL filter coffee = 200  
 $\therefore$  Coffee decoction in 100 mL filter coffee =  $200600 \times 100$   
 $= 1003$   
 $= 3313 \text{ mL}$   
 Since  $3313 > 30$ , this filter coffee is stronger.  
 S.No. 4. Here  $24 + 56 = 80$   
 $\therefore$  Coffee decoction in 80 mL filter coffee = 24 mL  
 $\therefore$  Coffee decoction in 100 mL filter coffee =  $2480 \times 100 = 30$   
 Since  $30 = 30$ , this filter coffee is regular.  
 S.No. 5. Here  $100 + 300 = 400$   
 $\therefore$  Coffee decoction in 400 mL filter coffee = 100 mL  
 $\therefore$  Coffee decoction in 100 mL filter coffee =  $100400 \times 100 = 25 \text{ mL}$   
 Since  $25 < 30$ , this filter coffee is lighter.

### Figure It Out (Pages 165-167)

**Question 1. Circle the following statements of proportion that are true.**

- (i)  $4 : 7 :: 12 : 21$
- (ii)  $8 : 3 :: 24 : 6$
- (iii)  $7 : 12 :: 12 : 7$



(iv)  $21 : 6 :: 35 : 10$

(v)  $12 : 18 :: 28 : 12$

(vi)  $24 : 8 :: 9 : 3$

**Solution:** (i) Given statement is  $4 : 7 :: 12 : 21$ .

This is true if  $4 \times 7 = 12 \times 21$

or if  $4 \times 7 = 28$ , which is true.

$\therefore$  The given statement is true.

(ii) Given statement is  $8 : 3 :: 24 : 6$ .

This is true if  $8 \times 3 = 24 \times 6$

or if  $8 \times 3 = 24$ , which is false.

$\therefore$  The given statement is not true.

(iii) Given statement is  $7 : 12 :: 12 : 7$ .

This is true if  $7 \times 12 = 12 \times 7$ , which is false.

$\therefore$  The given statement is not true.

(iv) Given statement is  $21 : 6 :: 35 : 10$ .

This is true if  $21 \times 6 = 35 \times 10$

or if  $21 \times 6 = 210$ , which is true.

$\therefore$  The given statement is true.

(v) Given statement is  $12 : 18 :: 28 : 12$ .

This is true if  $12 \times 18 = 28 \times 12$

or if  $2 \times 3 = 7 \times 3$

or  $2 = 7$ , which is false.

$\therefore$  The given statement is not true.

(vi) Given statement is  $24 : 8 :: 9 : 3$ .

This is true if  $24 \times 8 = 9 \times 3$

Or if  $3 = 3$ , which is true.

$\therefore$  The given statement is true.

**Question 2. Give 3 ratios that are proportional to  $4 : 9$ .**

\_\_\_\_\_ : \_\_\_\_\_ : \_\_\_\_\_ : \_\_\_\_\_

**Solution:** The given ratio is  $4 : 9$ .

We have  $4 \times 9 = 4 \times 2 \times 9 = 4 \times 3 \times 9 = 4 \times 4 \times 9$

or  $4 \times 9 = 8 \times 9 = 12 \times 9 = 16 \times 9$

$\therefore 4 : 9 :: 8 : 18, 4 : 9 :: 12 : 27$ , and  $4 : 9 :: 16 : 36$

$\therefore$  Ratios  $8 : 18, 12 : 27$  and  $16 : 36$  are proportional to the given ratio  $4 : 9$ .

**Question 3. Fill in the missing numbers for these ratios that are proportional to  $18 : 24$ .**

(i)  $3 : \underline{\hspace{2cm}}$

(ii)  $12 : \underline{\hspace{2cm}}$

(iii)  $20 : \underline{\hspace{2cm}}$



(iv) 27 : \_\_\_\_\_

**Solution:**

(i) Given ratio is 18 : 24.

Let  $18 : 24 :: 3 : x$

$$\therefore \frac{18}{24} = \frac{3}{x} \quad \text{or} \quad \frac{3}{4} = \frac{3}{x} \quad \text{or} \quad x = 4$$

$\therefore$  Missing number in the ratio 3 : \_\_\_\_\_ is **4**.

(ii) Let  $18 : 24 :: 12 : x$ .

$$\therefore \frac{18}{24} = \frac{12}{x} \quad \text{or} \quad \frac{3}{4} = \frac{12}{x}$$

$$\text{or} \quad 3x = 48 \quad \text{or} \quad x = \frac{48}{3} = 16$$

$\therefore$  Missing number in the ratio 12 : \_\_\_\_ is **16**.

(iii) Let  $18 : 24 :: 20 : x$ .

$$\therefore \frac{18}{24} = \frac{20}{x} \quad \text{or} \quad \frac{3}{4} = \frac{20}{x}$$

$$\text{or} \quad 3x = 80 \quad \text{or} \quad x = \frac{80}{3}$$

$\therefore$  Missing number in the ratio 20 : \_\_\_\_ is  **$\frac{80}{3}$** .

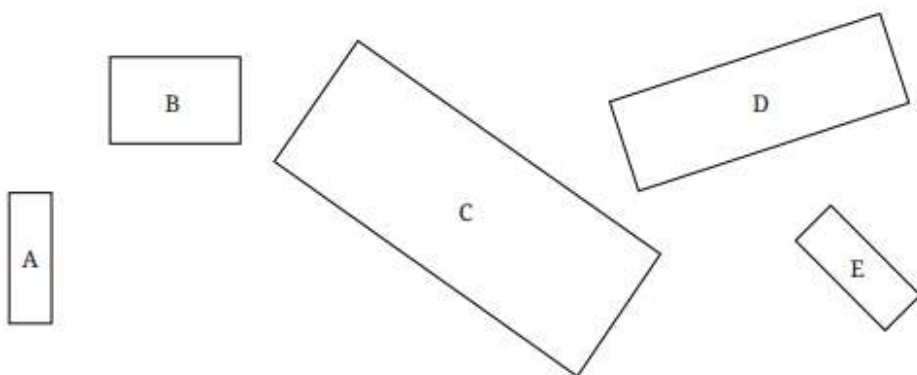
(iv) Let  $18 : 24 :: 27 : x$ .

$$\therefore \frac{18}{24} = \frac{27}{x} \quad \text{or} \quad \frac{3}{4} = \frac{27}{x}$$

$$\text{or} \quad 3x = 108 \quad \text{or} \quad x = \frac{108}{3} = 36$$

$\therefore$  Missing number in the ratio 27 : \_\_\_\_ is **36**.

**Question 4.** Look at the following rectangles. Which rectangles are similar to each other? You can verify this by measuring the width and height using a scale and comparing their ratios.



**Solution:** Using a scale, we measure the width and height of given rectangles.

Here, the ratio 'Width : Height' for given rectangles A, B, C, D, and E are respectively 1 : 3, 3 : 2, 9 : 4, 7 : 2 and 3 : 1.

These ratios are all distinct. The ratios of A and E are 1 : 3 and 3 : 1 respectively.



Rect-angle	Width (in mm)	Height (in mm)	Ratio = $\frac{\text{Width}}{\text{Height}}$
A	5	15	$\frac{5}{15} = \frac{1}{3}$
B	15	10	$\frac{15}{10} = \frac{3}{2}$
C	45	20	$\frac{45}{20} = \frac{9}{4}$
D	35	10	$\frac{35}{10} = \frac{7}{2}$
E	15	5	$\frac{15}{5} = \frac{3}{1}$

∴ For A and E, one side is 3 times the other side.

∴ Only rectangles A and E are similar.

**Question 5.** Look at the following rectangle. Can you draw a smaller rectangle and a bigger rectangle with the same width-to-height ratio in your notebook? Compare your rectangles with your classmates' drawings. Are all of them the same? If they are different from yours, can you think why? Are they wrong?



Solution:

For the given rectangle;

Width = 32 mm and height = 18 mm

∴ Ratio is 32 : 18.

We shall draw smaller and bigger rectangles and similar to the given rectangle by considering different 'factors of change'.

Let the factor of change be 12.

∴ New width =  $12 \times 32 = 16$  mm

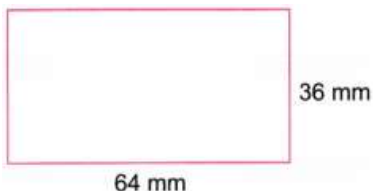
and New height =  $12 \times 18 = 9$  mm

A new, similar rectangle is shown in the figure.



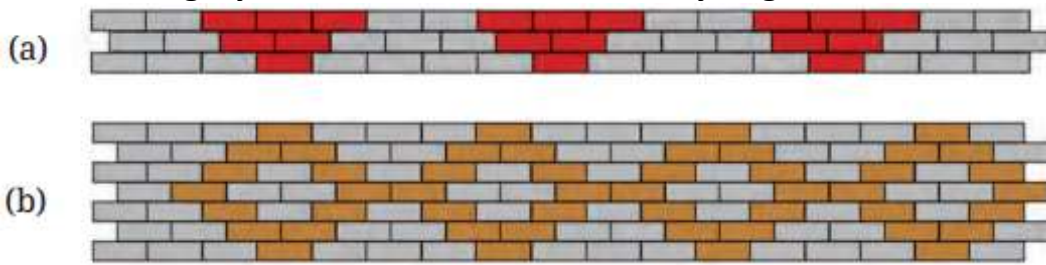
Let 'factor of change' be 2.

∴ New width =  $2 \times 32 = 64$  mm and new height =  $2 \times 18 = 36$  mm

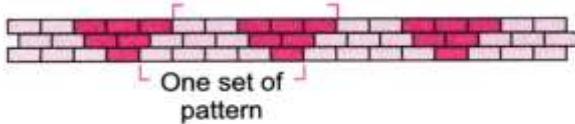


A new, similar rectangle is shown in the figure. The rectangles drawn by other classmates are all different, but they are all similar to the given rectangle.

**Question 6.** The following figure shows a small portion of a long brick wall with patterns made using coloured bricks. Each wall continues this pattern throughout the wall. What is the ratio of grey bricks to coloured bricks? Try to give the ratios in their simplest form.



**Solution:** (a) We consider one set of patterns in the given wall.



Number of grey bricks in one set of pattern =  $2 + 3 + 4 = 9$

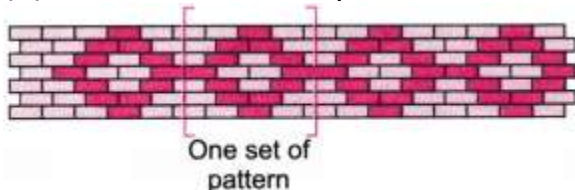
Number of coloured bricks in one set of pattern =  $3 + 2 + 1 = 6$

$\therefore$  Ratio of grey bricks to coloured bricks =  $9 : 6$

We have  $9 : 6 = 3 : 2$

$\therefore$  Ratio in the simplest form =  $3 : 2$

(b) We use one set of patterns on the given wall



Number of grey bricks in one set of pattern

$$= \left( \frac{1}{2} + 1 + 1 + \frac{1}{2} \right) + (1 + 1) + \left( \frac{1}{2} + 1 + \frac{1}{2} \right) + (1 + 1) + \left( \frac{1}{2} + 1 + \frac{1}{2} \right) + (1 + 1) + \left( \frac{1}{2} + 1 + \frac{1}{2} \right)$$

$$= 3 + 2 + 2 + 2 + 2 + 2 + 3$$

$$= 16$$

Number of coloured bricks in one set of pattern =  $1 + (1 + 1) + (1 + 1) + (1 + 1) + (1 + 1) + (1 + 1) + 1$

$$= 1 + 2 + 2 + 2 + 2 + 2 + 1$$

$$= 12$$

$\therefore$  Ratio of grey bricks to coloured bricks =  $16 : 12$

We have  $16 : 12 = 4 : 3$

$\therefore$  Ratio in the simplest form =  $4 : 3$ .

### Trairasika – The Rule of Three

**Example 8.** For the mid-day meal in a school with 120 students, the cook usually makes 15 kg of rice. On a rainy day, only 80 students came to school. How many kilograms of rice should the cook make so that the food is not wasted?

**The ratio of the number of students to the amount of rice needs to be proportional. So,  $120 : 15 :: 80 : ?$**

**What is the factor of change in the first term? (Page 167)**

**Solution:** For 120 students, the rice required is 15 kg.

Let  $x$  kg of rice be required for 80 students.

$\therefore$  Ratios  $120 : 15$  and  $80 : x$  are in proportion.

$\therefore 120 : 15 :: 80 : x$

$\Rightarrow 120 \times 15 = 80x$

$\Rightarrow x = 10$

$\therefore$  10 kg of rice is required.

Also, factor of change in the first term  $= \frac{80}{120} = \frac{2}{3}$

Alternative Method:

Factor of change in second term  $= x \times 15$

Since the quality of food is the same, we have  $\frac{2}{3} = x \times 15$

$\Rightarrow 3x = 30$

$\Rightarrow x = 10$

$\therefore$  10 kg of rice is required.

**Example 9. (i) A car travels 90 km in 150 minutes. If it continues at the same speed, what distance will it cover in 4 hours?**

**If it continues at the same speed, the ratio of the time taken should be proportional to the ratio of the distance covered.**

**(ii)  $150 : 90 :: 4 : x$**

**Is this the right way to formulate the question?**

**(iii) How can you find the distance covered in 240 minutes? (Page 169)**

**Solution:** (i) We have, 4 hours  $= 4 \times 60 = 240$  minutes

In 150 minutes, the distance covered  $= 90$  km

Let  $x$  km be covered in 4 hours, i.e., in 240 minutes.

$\therefore$  The ratios  $150 : 90$  and  $240 : x$  are in proportion.

$\therefore 150 : 90 :: 240 : x$

(ii) Since units must be the same in comparing ratios, the given proportion  $150 : 90 :: 4 : ?$  is meaningless.

We have 4 hours  $= 240$  minutes

$\therefore$  The proportion  $150 : 90 :: 240 : ?$  is correct.

(iii) We have  $150 : 90 :: 240 : x$

$\Rightarrow \frac{150}{90} = \frac{240}{x}$

$\Rightarrow 53 = 240x$

$\Rightarrow 5x = 3 \times 240$

$\Rightarrow x = 144$

$\therefore$  Distance covered in 4 hours  $= 144$  km.

**Example 10.** A small farmer in Himachal Pradesh sells each 200 g packet of tea for ₹ 200. A large estate in Meghalaya sells each 1 kg packet of tea for ₹ 800. Are the weight-to-price ratios in both places proportional? Which tea is more expensive? Why? (Page 169)

**Solution:** We have 1 kg = 1000 g

In Himachal Pradesh, 200 g of tea costs ₹ 200.

∴ Ratio of weight to price in Himachal Pradesh = 200 : 200 = 1 : 1

In Meghalaya, 1000 g tea costs ₹ 800.

∴ Ratio of weight to price in Meghalaya = 1000 : 800 = 5 : 4

The ratios 1 : 1 and 5 : 4 are not proportional, because  $1 \neq 5/4$

Price of 200 g tea in Himachal Pradesh = ₹ 200

Price of 1000 g tea in Himachal Pradesh =  $200/200 \times 1000 = ₹ 1000$

Since  $1000 > 800$ , tea is more expensive in Himachal Pradesh.

### Figure It Out (Pages 170-171)

**Question 1.** The Earth travels approximately 940 million kilometres around the Sun in a year. How many kilometres will it travel in a week?

**Solution:** We know that:

1 million = 10 lakh = 10,00,000

and 1 year = 365 weeks.

940 million kilometres, i.e.,  $940 \times 10,00,000$  kilometres, are travelled by the Earth in 1 year, i.e., in 365 weeks.

Let the Earth travel  $x$  kilometres in 1 week

∴ The ratios  $940 \times 10,00,000 : 365$  and  $x : 1$  are in proportion.

$$\Rightarrow 940 \times 10,00,000/365 = x/1$$

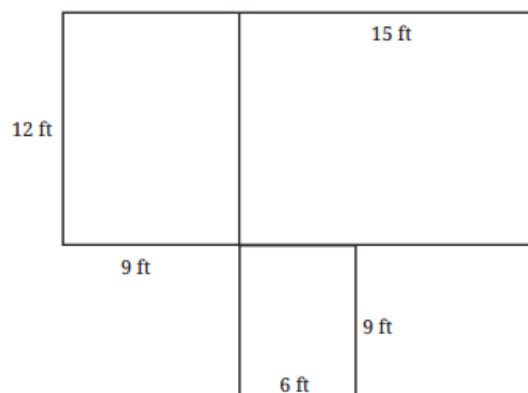
$$\Rightarrow x = 940 \times 10,00,000/365$$

$$\Rightarrow x = 2575315.068$$

$$\Rightarrow x = 2,57,53,151 \text{ (nearly)}$$

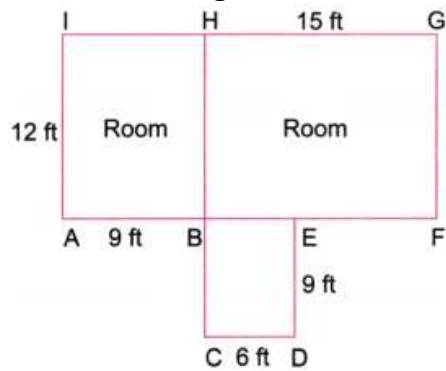
∴ In 1 week, Earth travels nearly 2,57,53,151 kilometres around the Sun.

**Question 2.** A mason is building a house in the shape shown in the diagram. He needs to construct both the outer walls and the inner wall that separates the two rooms. To build a wall of 10 feet, he requires approximately 1450 bricks. How many bricks would he need to build the house? Assume all walls are of the same height and thickness.



**Solution:** Number of bricks required for a 10 ft wall = 1450

∴ Ratio of length of wall to number of bricks = 10 : 1450



Total length of walls = AI + CH + DE + FG + IG + AF + CD  
= 12 + (9 + 12) + 9 + 12 + (9 + 15) + (9 + 15) + 6  
= 108 ft

Let x bricks be required for a 108 ft long wall.

∴ Ratio of length of wall to number of bricks = 108 : x

These ratios are in proportion.

∴ 10 : 1450 :: 108 : x

⇒  $10 \times 1450 = 108x$

⇒  $1450 = 108x$

⇒  $x = 145 \times 108 = 15,660$

∴ Number of required bricks = 15,660.

**Question 3.** Puneeth's father went from Lucknow to Kanpur in 2 hours by riding his motorcycle at a speed of 50 km/h. If he drives at 75 km/h, how long will it take him to reach Kanpur? Can we form this problem as a proportion 50 : 2 :: 75 : \_\_\_\_

Would it take Puneeth's father more time or less time to reach Kanpur? Think about it.

(Page 171)

**Solution:** Time taken at the speed of 50 km/h = 2 hours

∴ Distance = Speed × Time

= 50 × 2

= 100 km

At speed of 75 km/h, time taken =  $\frac{100}{75} = \frac{4}{3}$  hours

∴ Ratio of speed to time in both cases are 50 : 2 and 75 :  $\frac{4}{3}$

Here,  $\frac{50}{2} = 25$  and  $\frac{75}{\frac{4}{3}} = 75 \times \frac{3}{4} = \frac{225}{4} \neq 25$

∴ The ratios 50 : 2 and 75 :  $\frac{4}{3}$  are not in proportion.

∴ We cannot write 50 : 2 :: 75 :  $\frac{4}{3}$ .

At a speed of 75 km/h, Puneeth's father will take  $\frac{4}{3}$  hours to reach Kanpur, which is less than 2 hours.

7.5 Sharing, But Not Equally

**Example 11.** Prashanti and Bhuvan started a food cart business near their school.

Prashanti invested ₹ 75,000 and Bhuvan invested ₹ 25,000. At the end of the first month, they gained a profit of ₹ 4,000. They decided that they would share the profit in the same ratio as their investment. What is each person's share of the profit? (Page 174)

**Solution:** Investment of Prashanti = ₹ 75,000

Investment of Bhuvan = ₹ 25,000

∴ Ratio of investment = 75000 : 25000 = 3 : 1

Total profit = ₹ 4,000

∴ Share of Prashanti =  $33+1 \times 4,000$

=  $34 \times 4,000$

= ₹ 3,000

∴ Share of Bhuvan =  $13+1 \times 4,000$

=  $14 \times 4,000$

= ₹ 1,000

Verification: ₹ 3,000 + ₹ 1,000 = ₹ 4,000, the total profit.

**Example 12.** A mixture of 40 kg contains sand and cement in the ratio of 3 : 1. How much cement should be added to the mixture to make the ratio of sand to cement 5 : 2? (Page 174)

**Solution:** Ratio of sand and cement = 3 : 1

Weight of mixture = 40 kg

∴ Weight of sand in mixture =  $33+1 \times 40$

=  $34 \times 40 = 30$  kg

Weight of cement in the mixture =  $13+1 \times 40$

=  $14 \times 40 = 10$  kg

Let the weight of cement in the new mixture be x kg, so that the new ratio is 5 : 2.

∴ Ratios 30 : x and 5 : 2 are in proportion.

⇒  $30x = 52$

⇒  $5x = 60$

⇒  $x = 12$

Weight of cement added = 12 kg – 10 kg = 2 kg

**Figure It Out (Page 175)**

**Question 1.** Divide ₹ 4,500 into two parts in the ratio 2 : 3.

**Solution:** Given ratio = 2 : 3

Amount to be divided = ₹ 4,500

∴ First part =  $22+3 \times 4,500$

=  $25 \times 4,500$

=  $2 \times 900$

= ₹ 1,800

∴ Second part =  $32+3 \times 4,500$

=  $35 \times 4,500$

=  $3 \times 900$

= ₹ 2,700

∴ Two parts are ₹ 1,800 and ₹ 2,700.

Verification:

$$1,800 : 2,700 = 1,800 \div 2,700 = 1800 \div 2700 = 23 = 2 : 3 \text{ and } 1,800 + 2,700 = 4,500.$$

**Question 2. In a science lab, acid and water are mixed in the ratio of 1 : 5 to make a solution. In a bottle that has 240 mL of the solution, how much acid and water does the solution contain?**

**Solution:** Ratio of acid and water = 1 : 5

Quantity of solution = 240 mL

$$\therefore \text{Quantity of acid} = \frac{1}{1+5} \times 240$$

$$= \frac{1}{6} \times 240$$

$$= 40 \text{ mL}$$

$$\therefore \text{Quantity of water} = \frac{5}{1+5} \times 240$$

$$= \frac{5}{6} \times 240$$

$$= 200 \text{ mL}$$

$\therefore$  Quantities of acid and water in the solution are 40 mL and 200 mL.

Verification:

$$40 : 200 = 40 \div 200 = 15 = 1 : 5 \text{ and } 40 + 200 = 240.$$

**Question 3. Blue and yellow paints are mixed in the ratio of 3 : 5 to produce green paint. To produce 40 mL of green paint, how much of these two colours are needed? To make the paint a lighter shade of green, I added 20 mL of yellow to the mixture. What is the new ratio of blue and yellow in the paint?**

**Solution:** Ratio of blue and yellow paints = 3 : 5

Quantity of green paint = 40 mL

$$\therefore \text{Quantity of blue paint} = \frac{3}{3+5} \times 40$$

$$= \frac{3}{8} \times 40$$

$$= 15 \text{ mL}$$

$$\therefore \text{Quantity of yellow paint} = \frac{5}{3+5} \times 40$$

$$= \frac{5}{8} \times 40$$

$$= 25 \text{ mL}$$

Addition of yellow paint to the mixture = 20 mL

$$\therefore \text{New quantity of blue paint} = 15 \text{ mL}$$

$$\therefore \text{New quantity of yellow paint} = 25 \text{ mL} + 20 \text{ mL} = 45 \text{ mL}$$

$$\therefore \text{New ratio of blue and yellow paints} = 15 : 45 = 1 : 3.$$

**Question 4. To make soft idlis, you need to mix rice and urad dal in the ratio of 2 : 1. If you need 6 cups of this mixture to make idlis tomorrow morning, how many cups of rice and urad dal will you need?**

**Solution:** Ratio of rice and urad dal = 2 : 1

Total number of cups of mixture = 6

$$\therefore \text{Number of cups of rice} = \frac{2}{2+1} \times 6$$

$$= \frac{2}{3} \times 6$$

$$= 4$$

$$\therefore \text{Number of cups of urad dal} = \frac{1}{2+1} \times 6$$

$$= \frac{1}{3} \times 6$$





= 2

∴ 4 cups of rice and 2 cups of urad dal are to be mixed.

**Question 5.** I have one bucket of orange paint that I made by mixing red and yellow paints in the ratio of 3 : 5. I added another bucket of yellow paint to this mixture. What is the ratio of red paint to yellow paint in the new mixture?

**Solution:** Let the capacity of one bucket be x L.

Ratio of red paint and yellow paint = 3 : 5

∴ Quantity of red paint in the bucket =  $3 \times 5x = 3x8$

∴ Quantity of yellow paint in the bucket =  $5 \times 5x = 5x8$

One bucket of yellow paint is added to the mixture.

∴ New quantity of red paint in the mixture =  $3x8$

∴ New quantity of yellow paint in the mixture =  $5x8 + x = 13x8$

∴ New ratio of red paint and yellow paint in the mixture =  $3x8 : 13x8 = 3 : 13$

7.6 Unit Conversions

### Figure It Out (Pages 176-177)

**Question 1.** Anagh mixes 600 mL of orange juice with 900 mL of apple juice to make a fruit drink. Write the ratio of orange juice to apple juice in its simplest form.

**Solution:** Quantity of orange juice = 600 mL

Quantity of apple juice = 900 mL

∴ Ratio of orange juice to apple juice = 600 : 900

Ratio in the simplest form =  $600 : 900 = 2 : 3$

**Question 2.** Last year, we hired 3 buses for the school trip. We had a total of 162 students and teachers who went on that trip, and all the buses were full. This year we have 204 students. How many buses will we need? Will all the buses be full?

**Solution:** Number of buses for 162 students and teachers = 3

Since the buses were full, the capacity of 1 bus =  $162 \div 3 = 54$

∴ Ratio of number of seats to the number of buses is 54 : 1.

We have

$54 : 1 = 2(54) : 2(1) = 108 : 2$

$54 : 1 = 3(54) : 3(1) = 162 : 3$

$54 : 1 = 4(54) : 4(1) = 216 : 4$

∴ Capacity of 4 buses = 216

∴ For 204 students, we shall need 4 buses.

Since  $216 - 204 = 12$ , we have 12 vacant seats in the buses.

**Question 3.** The area of Delhi is 1,484 sq. km, and the area of Mumbai is 550 sq. km. The population of Delhi is approximately 30 million, and that of Mumbai is 20 million people. Which city is more crowded? Why do you say so?

**Solution:** Area of Delhi = 1,484 sq.km

Population of Delhi = 30 million

Area of Mumbai = 550 sq. km

Population of Mumbai = 20 million

∴ Ratio of area to population for Delhi = 1484 : 30

∴ Ratio of area to population for Mumbai = 550 : 20

Factor of change of area =  $\frac{550}{1484} = 0.371$  (nearly)

Factor of change of population =  $\frac{20}{30} = 0.667$  (nearly)

Since  $0.667 > 0.371$ , Mumbai is more crowded than Delhi.

Alternative Method:

Ratio of area to population for Delhi = 1484 : 30

Let the density of Delhi and Mumbai be the same, and there be  $x$  people in Mumbai.

∴ The ratios 1,484 : 30 and 550 :  $x$  are in proportion.

∴  $1,484 \times 30 = 550x$

$\Rightarrow 1484x = 30 \times 550 = 16,500$

$\Rightarrow x = \frac{16500}{1484} = 11.118$

There should be 11.118 million people in Mumbai. But the population of Mumbai is 20 million.

∴ Mumbai is more crowded than Delhi.

**Question 4.** A crane of height 155 cm has its neck and the rest of its body in the ratio 4 : 6. For your height, if your neck and the rest of the body also had this ratio, how tall would your neck be?



**Solution:** The ratio of the height of the neck and the height of the rest of the body of a crane is 4 : 6.

My height is 65 inches, i.e., 165 cm.

Let the ratio of the height of my neck and the height of the rest of my body also be 4 : 6.

∴ Height of my neck =  $(\frac{4}{10} \times 165)$  cm = 66 cm

**Question 5.** Let us try an ancient problem from Lilavati. At that time, weights were measured in a unit named palas, and niskas was a unit of money. "If 212 palas of saffron costs 37 niskas, O expert businessman! Tell me quickly, what quantity of saffron can be bought for 9 niskas?"

**Solution:** Here, the unit of weight in palas and the unit of money are niskas.

Cost of 212 palas of saffron = 37 niskas

∴ Ratio of weight to price is 212 : 37

or 52 : 37

or 35 : 6

Let  $x$  palas of saffron be bought for 9 niskas.

∴ Ratio of weight to price is  $x : 9$ .

These ratios are in proportion.

$$\therefore 35 : 6 :: x : 9$$

$$\Rightarrow 35 \times 9 = 6x$$

$$\Rightarrow 6x = 35 \times 9$$

$$\Rightarrow x = 52.5$$

∴ 52.5 palas of saffron can be bought for 9 niskas.

**Question 6.** Harmain is a 1-year-old girl. Her elder brother is 5 years old. What will be Harmain's age when the ratio of her age to her brother's age is 1 : 2?

**Solution:** The ages of Harmain and her brother are 1 year and 5 years.

Let  $x$  years, the ratio of their ages be 1 : 2.

Age of Harmain after  $x$  years =  $(1 + x)$  years.

Age of her brother after  $x$  years =  $(5 + x)$  years.

∴ After  $x$  years, ratio of their ages =  $1 + x : 5 + x$

The ratios are in proportion.

$$\therefore 1 : 2 :: 1 + x : 5 + x$$

$$\Rightarrow 12 = 1 + x5 + x$$

$$\Rightarrow 5 + x = 2(1 + x)$$

$$\Rightarrow 5 + x = 2 + 2x$$

$$\Rightarrow 2x - x = 5 - 2$$

$$\Rightarrow x = 3$$

∴ After 3 years, the age of Harmain =  $1 + 3 = 4$  years.

Verification:

After 3 years, age of her brother =  $5 + 3 = 8$  years

Also, the ratio of their ages =  $4 : 8 = 1 : 2$ .

**Question 7.** The mass of equal volumes of gold and water is in the ratio 37 : 2. If 1 litre of water is 1 kg in mass, what is the mass of liter of gold?

**Solution:** The ratio of masses of gold and water, when their volumes are the same, is 37 : 2.

Mass of 1 litre of water = 1 kg

Let the mass of 1 litre of gold =  $x$  kg

∴ With equal volumes, the ratio of masses of gold and water is  $x : 1$ .

These ratios are in proportion.

$$\therefore 37 : 2 :: 1 : x$$

$$\Rightarrow 37 \times 2 = x1$$

$$\Rightarrow x = 372$$

Thus, the mass of 1 litre of gold is 372 kg.

**Question 8.** It is good farming practice to apply 10 tonnes of cow manure for 1 acre of land. A farmer is planning to grow tomatoes in a plot of size 200 ft by 500 ft. How much manure should he buy? (Please refer to the section on Unit Conversions earlier in this chapter).

**Solution:** We have 1 ton = 1,000 kg

∴ 10 tonnes =  $10 \times 1,000 = 10,000$  kg

Also, 1 acre = 43,560 sq. ft.

∴ Ratio of cow manure to area of land in kg and sq. ft. = 10,000 : 43,560

Size of plot = 200 ft. by 500 ft.

∴ Area of plot =  $200 \times 500 = 1,00,000$  sq. ft.

Let cow manure be x kg.

∴ Ratio of cow manure to area of plot =  $x : 1,00,000$

These ratios are in proportion.

∴  $10,000 : 43,560 :: x : 1,00,000$

$\Rightarrow 10,000 \times 43,560 = x \times 1,00,000$

$\Rightarrow 43,560x = 10,000 \times 1,00,000 = 1,00,00,00,000$

$\Rightarrow x = \frac{1,00,00,00,000}{43,560}$

$\Rightarrow x = 22956.84$

∴ Required cow manure = 22956.84 kg = 22.95684 tonnes.

**Question 9. A tap takes 15 seconds to fill a mug of water. The volume of the mug is 500 mL. How much time does the same tap take to fill a bucket of water if the bucket has a 10-litre capacity?**

**Solution:** Time taken by the tap for 500 mL of water = 15 seconds

∴ Ratio of volume to time = 500 : 15

We know 1 litre = 1,000 mL

10 litre =  $10 \times 1,000 = 10,000$  mL

Let the time taken to fill a bucket of 10,000 mL be x seconds.

∴ Ratio of volume to time = 10,000 : x

These ratios are proportional.

∴  $500 : 15 :: 10,000 : x$

$\Rightarrow 500 \times 15 = 10,000x$

$\Rightarrow 500x = 1,50,000$

$\Rightarrow x = 300$

∴ Time to fill bucket = 300 seconds

= 300/60 minutes

= 5 minutes.

**Question 10. One acre of land costs ₹ 15,00,000. What is the cost of 2,400 square feet of the same land?**

**Solution:** We know that 1 acre = 43,560 square feet.

∴ Cost of 43,560 sq. ft. land = ₹ 15,00,000

∴ Ratio of area of land to cost = 43,560 : 15,00,000

Let the cost of 2,400 sq. ft. of land be ₹ x.

∴ Ratio of area of land to cost = 2,400 : x

These ratios are proportional.

∴  $43,560 : 15,00,000 :: 2,400 : x$

$\Rightarrow 43,560 \times 15,00,000 = 2,400x$

$\Rightarrow 43,560x = 2,400 \times 15,00,000$

$\Rightarrow x = \frac{2,400 \times 15,00,000}{43,560}$

$$\Rightarrow x = 82,664.63$$

$\therefore$  Cost of land = ₹ 82,664.63.

**Question 11.** A tractor can plough the same area of a field 4 times faster than a pair of oxen. A farmer wants to plough his 20-acre field. A pair of oxen takes 6 hours to plough an acre of land. How much time would it take if the farmer used a pair of oxen to plough the field? How much time would it take him if he decides to use a tractor instead?

**Solution:** Ratio of efficiency of a tractor to a pair of oxen = 4 : 1

Time taken by a pair of oxen to plough 1 acre of field = 6 hours

$\therefore$  Time taken by a tractor to plough 1 acre field =  $6 \div 4 = 1.5$  hours

$\therefore$  Time taken by a pair of oxen to plough 20-acre field =  $20 \times 6 = 120$  hours

$\therefore$  Time taken by a tractor to plough a 20-acre field =  $20 \times 1.5 = 30$  hours

**Question 12.** The ₹ 10 coin is an alloy of copper and nickel called 'cupro-nickel'. Copper and nickel are mixed in a 3 : 1 ratio to get this alloy. The mass of the coin is 7.74 grams. If the cost of copper is ₹ 906 per kg and the cost of nickel is ₹ 1,341 per kg, what is the cost of these metals in a ₹ 10 coin?

**Solution:**

Ratio of copper and nickel in ₹ 10 coin = 3 : 1

Mass of one ₹ 10 coin = 7.74 grams

$\therefore$  Mass of copper in one ₹ 10 coin =  $3 \div 4 \times 7.74$

=  $3 \times 7.74$

= 5.805 grams

Mass of nickel in one ₹ 10 coin =  $1 \div 4 \times 7.74$

=  $1 \times 7.74$

= 1.935 grams

Cost of 1 kg copper = ₹ 906

$\therefore$  Cost of 1000 grams of copper = ₹ 906

$\therefore$  Cost of 5.805 grams copper =  $906 \div 1000 \times 5.805 = ₹ 5.26$

Cost of 1 kg nickel = ₹ 1341

$\therefore$  Cost of 1000 grams of nickel = ₹ 1341

$\therefore$  Cost of 1.935 grams nickel =  $1341 \div 1000 \times 1.935 = ₹ 2.59$

$\therefore$  In one ₹ 10 coin, the cost of copper and the cost of nickel are respectively ₹ 5.26 and ₹ 2.59.